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r-neutrosophic Subset of G-submodules

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Abstract

This article explains a particular category of neutrosophic subsets of G-submodules, specifically r-neutrosophic subsets, where $r \in [0, 1]$. The algebra of r-neutrosophic subsets of G-submodules is discussed, along with some fundamental characteristics of their sum. Definitions and theorems related to this concept are provided to clarify the properties of an arbitrary non-empty family of r-neutrosophic subsets of G-submodules.

Keywords: Neutrosophic set, Neutrosophic G-submodule , r neutrosophic subset of G-submodule, Neu-trosophic point

1. Introduction

The key factors of indeterminacy, uncertainty, and true values in real-valued problems have led to the development of a new set theory known as the neutrosophic set developed by Florentin Smarandache [1–3]. The neutrosophic set extends fuzzy [4] and intuitionistic fuzzy sets [5] by introducing three independent membership functions for each element: truth-membership, falsehood-membership, and indeterminacy-membership. One of the major research areas in the current scenario is the generalization or expansion of classical algebraic structures within the neutrosophic domain, which is used for modeling real-world problems.

The algebraic structure of a G-module, introduced by Curtis [6], generalizes the action of a group G on a vector space M, helping to further refine the study of group representation. Fuzzy G-modules and intuitionistic fuzzy G-modules, introduced by Sherry Fernandez [7, 8] and Sharma et al. [9–12] respectively , are generalizations

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of G-modules, forming a new type of generalized algebraic structure. Some characterization and structural properties of neutrosohic G-modules are studied by Binu and P.Isaac [13, 14]. This study presents a special class of neutrosophic sets of G-submodules, referred to as r-neutrosophic G-submodules, where $r \in [0, 1]$. Additionally, this paper explores the algebra of r-neutrosophic set of G-submodules, including concepts of sum in various contexts and their algebraic properties in arbitrary collection.

2. Preliminary Concepts

We provide some of the fundamental concepts and results in this session, which are necessary for a better comprehension of the sessions that follow.

Definition 2.1. [15] "Let $(G, *)$ be a group. A vector space M over the field K is called a *G-module*, denoted as G_M, if for every $g \in G$ and $m \in M$; $\exists$ a product (called the action of G on M), $g \cdot m \in M$ satisfies the following axioms

(1) $1_G \cdot m = m; \quad \forall m \in M \quad$ (1_G being the identity element of G)
(2) $(g * h) \cdot m = g \cdot (h \cdot m); \quad \forall m \in M$ and $g, h \in G$
(3) $g \cdot (k_1 m_1 + k_2 m_2) = k_1(g \cdot m_1) + k_2(g \cdot m_2); \forall\ k_1, k_2 \in K; m_1, m_2 \in M$ ".

Example 2.1. [8] "Let $G = \{1, -1, i, -i\}$ and $M = \mathbb{C}^n$; $(n \geq 1)$. Then M is a vector space over $\mathbb{C}$ and under the usual addition and multiplication of complex numbers we can show that M is a G-module ".

Definition 2.2. [2] "A *neutrosophic set* P of the universal set X is defined as $P = \{(x, t_P(x), i_P(x), f_P(x)) : x \in X\}$ where $t_P, i_P, f_P : X \to (^-0, 1^+)$. The three components t_P, i_P *and* f_P represent membership value (Percentage of truth), indeterminacy (Percentage of indeterminacy) and non membership value (Percentage of falsity) respectively. These components are functions of non standard unit interval $(^-0, 1^+)$ ".

Remark 2.1. [16] "If $t_P, i_P, f_P : X \to [0, 1]$, then P is known as single valued neutrosophic set (SVNS)"

Definition 2.3. [2] "Let P and Q be two neutrosophic sets of X. Then P is contained in Q, denoted as $P \subseteq Q$ if and only if $P(x) \leqslant Q(x)\ \forall x \in X$, this means that $t_P(x) \leq t_Q(x),\ i_P(x) \leq i_Q(x),\ f_P(x) \geq f_Q(x),\ \forall\ x \in X$ ".

Definition 2.4. [2] " Let $P, Q \in U^X\ \forall\ x \in X$. Then

1. The union C of P and Q is denoted by $C = P \cup Q$ and defined as $C(x) = P(x) \vee Q(x)$ where $C(x) = \{x, t_C(x), i_C(x), f_C(x) : x \in X\}$ is given by

$$t_C(x) = t_P(x) \vee t_Q(x)$$
$$i_C(x) = i_P(x) \vee i_Q(x)$$
$$f_C(x) = f_P(x) \wedge f_Q(x)$$

2. The intersection C of P and Q is denoted by $C = P \cap Q$ and is defined as $C(x) = P(x) \wedge Q(x)$ where $C(x) = \{x, t_C(x), i_C(x), f_C(x) : x \in X\}$ is given by

$$t_C(x) = t_P(x) \wedge t_Q(x)$$
$$i_C(x) = i_P(x) \wedge i_Q(x)$$
$$f_C(x) = f_P(x) \vee f_Q(x) \text{ ''.}$$

Definition 2.5. [17] "Let $P_i,\ i \in J$ be an arbitrary non empty family of neutrosophic sets in U^X, then

$\bigcap_{i \in J} P_i = \{x, t_{\bigcap_{i \in J} P_i}(x), i_{\bigcap_{i \in J} P_i}(x), f_{\bigcap_{i \in J} P_i}(x) : x \in X\}$ where

$$t_{\bigcap_{i \in J} P_i}(x) = \underset{i \in J}{\wedge} t_{P_i}(x)$$
$$i_{\bigcap_{i \in J} P_i}(x) = \underset{i \in J}{\wedge} i_{P_i}(x)$$
$$f_{\bigcap_{i \in J} P_i}(x) = \underset{i \in J}{\vee} f_{P_i}(x)\text{''}$$

Definition 2.6. [18] "Let P and Q be neutrosophic sets of an R-Module M . Then their sum $P + Q$ is a neutrosophic set of M, defined as follows

$$P + Q(x) = \{x, t_{P+Q}(x), i_{P+Q}(x), f_{P+Q}(x) : x \in M\} \text{ where}$$
$$t_{P+Q}(x) = \vee\{t_P(y) \wedge t_Q(z) | x = y + z, y, z \in M\}$$
$$i_{P+Q}(x) = \vee\{i_P(y) \wedge i_Q(z) | x = y + z, y, z \in M\}$$
$$f_{P+Q}(x) = \wedge\ f_P(y) \vee f_Q(z) | x = y + z, y, z \in M\}\text{''}$$

Proposition 2.1. [19] If $A \in U(M)$ *and* $B \in U(N)$, then $A \times B \in U(M \times N)$.

Definition 2.7. [13] Let M be a G module over a field K and $A \in U^{G_M}$, where U^{G_M} denotes the set of all neutrosophic set of M. Then the neutrosophic set $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in M\}$ is said to be a neutrosophic G-module if the following conditions are satisfied:

(1) $t_A(ax + by) \geq t_A(x) \wedge t_A(y)$
$i_A(ax + by) \geq i_A(x) \wedge i_A(y)$
$f_A(ax + by) \leq f_A(x) \vee f_A(y),\ \forall\ x, y \in M, a, b \in K$

(2) $t_A(gm) \geq t_A(m)$
$i_A(gm) \geq i_A(m)$
$f_A(gm) \leq f_A(m)\ \forall\ g \in G, m \in M$

Remark 2.2. (1) We denote the all neutrosophic sets of G-submodules of M by $U(G_M)$.

Definition 2.8. [13] For any $x \in X$, the neutrosophic point $\hat{N}_{\{x\}}$ is defined as $\hat{N}_{\{x\}}(s) = \{(s, t_{\hat{N}_{\{x\}}}(s), i_{\hat{N}_{\{x\}}}(s), f_{\hat{N}_{\{x\}}}(s) : s \in X\}$ where

$$\hat{N}_{\{x\}}(s) = \begin{cases} (1,1,0) & x = s \\ (0,0,1) & x \neq s \end{cases}$$

Remark 2.3. Let X be a non empty set. The neutrosophic point $\hat{N}_{\{0\}}$ in X is $\hat{N}_{\{0\}}(x) = \{(x, t_{\hat{N}_{\{0\}}}(x), i_{\hat{N}_{\{0\}}}(x), f_{\hat{N}_{\{0\}}}(x) : x \in X\}$ where

$$\hat{N}_{\{0\}}(x) = \begin{cases} (1,1,0) & x = 0 \\ (0,0,1) & x \neq 0 \end{cases}$$

3. r-Neutrosophic subset of a neutrosophic- G submodule

Definition 3.1. Let $A = \{(x, t_A(x), i_A(x), f_A(x)) : x \in M\}$ be a neutrosophic G submodule of M over a field K ie, $A \in U(G_M)$. Then for each $r \in [0, 1]$, the r-neutrosophic subset of a neutrosophic- G submodule A of M is denoted and defined as $A_r = \{(x, t_{A_r}(x), i_{A_r}(x), f_{A_r}(x)) : x \in M\}$ where

$$t_{A_r}(x) = t_A(x) \wedge r,\ \ i_{A_r}(x) = i_A(x) \wedge r,\ \ f_{A_r}(x) = f_A(x) \vee 1 - r$$

Definition 3.2. Let $A_r,\ B_r \in U^{G_M}$. Then their sum $A_r + B_r \in U^{G_M}$ defined as follows

$$A_r + B_r(x) = \{x, t_{A_r+B_r}(x), i_{A_r+B_r}(x), f_{A_r+B_r}(x) : x \in M\}$$

where

$$t_{A_r+B_r}(x) = \vee\{t_{A_r}(y) \wedge t_{B_r}(z) | x = y + z, y, z \in M\}$$
$$i_{A_r+B_r}(x) = \vee\{i_{A_r}(y) \wedge i_{B_r}(z) | x = y + z, y, z \in M\}$$
$$f_{A_r+B_r}(x) = \wedge\{f_{A_r}(y) \vee f_{B_r}(z) | x = y + z, y, z \in M\}$$

Definition 3.3. For any $A_r \in U^{G_M}$, $\lambda A_r \in U^{G_M}$ defined as follows

$$\lambda A_r = \{x, t_{\lambda A_r}(x), i_{\lambda A_r}(x), f_{\lambda A_r}(x) : x \in M, \lambda \in K\}$$

where

$$t_{\lambda A_r}(x) = \begin{cases} \vee\{t_{A_r}(y)\} & if\ y \in M, x = \lambda y \\ 0 & otherwise \end{cases}$$

$$i_{\lambda A_r}(x) = \begin{cases} \vee\{i_{A_r}(y)\} & if\ y \in M, x = \lambda y \\ 0 & otherwise \end{cases}$$

$$f_{\lambda A_r}(x) = \begin{cases} \wedge\{f_{A_r}(y)\} & if\ y \in M, x = \lambda y \\ 0 & otherwise \end{cases}$$

Proposition 3.1. Let $A_r = \{(x, t_{A_r}(x), i_{A_r}(x), f_{A_r}(x)) : x \in M\} \in U^{G_M}$, then $t_{\lambda A_r}(\lambda x) \geq t_{A_r}(x), i_{\lambda A_r}(\lambda x) \geq i_{A_r}(x)$ and $f_{\lambda A_r}(\lambda x) \leq f_{A_r}(x)$.

Proof: We have

$$\lambda A_r = \{x, t_{\lambda A_r}(x), i_{\lambda A_r}(x), f_{\lambda A_r}(x) : x \in M, \lambda \in K\}$$

$$t_{\lambda A_r}(x) = \begin{cases} \vee\{t_{A_r}(y)\} & if\ y \in M, x = \lambda y \\ 0 & otherwise \end{cases}$$

$\Rightarrow,\ t_{\lambda A_r}(\lambda x) = \vee\{t_{A_r}(y), if\ y \in M, \lambda x = \lambda y\} \geq t_{A_r}(x)\ \forall x \in M$

Similarly $i_{\lambda A_r}(\lambda x) \geq i_{A_r}(x)$. Also

$$f_{\lambda A_r}(x) = \begin{cases} \wedge\{f_{A_r}(y)\} & if\ y \in M, x = \lambda y \\ 0 & otherwise \end{cases}$$

$\Rightarrow,\ f_{\lambda A_r}(\lambda x) = \wedge\{f_{A_r}(y), if\ y \in M, \lambda x = \lambda y\} \leq f_{A_r}(x)\ \forall x \in M$

Hence the proof. □

Theorem 3.1. If $A_r, B_r \in U^{G_M}$, then $\forall x, y \in M,\ \lambda, \nu \in K$, then

(1) $t_{(\lambda A_r + \nu B_r)}(\lambda x + \nu y) \geq t_{A_r}(x) \wedge t_{B_r}(y) \geq t_A(x) \wedge t_B(y)$
(2) $i_{(\lambda A_r + \nu B_r)}(\lambda x + \nu y) \geq i_{A_r}(x) \wedge i_{B_r}(y) \geq i_A(x) \wedge i_B(y)$
(3) $f_{(\lambda A_r + \nu B_r)}(\lambda x + \nu y) \leq f_{A_r}(x) \vee f_{B_r}(y) \leq f_A(x) \vee f_B(y)$

Proof: 1. We know

$$\begin{aligned} t_{(\lambda A_r + \nu B_r)}(\lambda x + \nu y) &= \bigvee\{t_{\lambda A_r}(\vartheta_1) \wedge t_{\nu B_r}(\vartheta_2) : \vartheta_1, \vartheta_2 \in M, \vartheta_1 + \vartheta_2 = \lambda x + \nu y\} \\ &\geq t_{\lambda A_r}(\lambda x) \wedge t_{\nu B_r}(\nu y) \\ &\geq t_{A_r}(x) \wedge t_{B_r}(y)\ \forall\ x, y \in M,\ \lambda, \nu \in K \\ &= (t_A(x) \wedge r) \wedge (t_B(y) \wedge r) \\ &\geq t_A(x) \wedge t_B(y) \end{aligned}$$

2. Same as above.
3. We know

$$\begin{aligned} f_{(\lambda A_r + \nu B_r)}(\lambda x + \nu y) &= \bigwedge\{f_{\lambda A_r}(\vartheta_1) \vee f_{\nu B_r}(\vartheta_2) : \vartheta_1, \vartheta_2 \in M, \vartheta_1 + \vartheta_2 = \lambda x + \nu y\} \\ &\leq f_{\lambda A_r}(\lambda x) \vee f_{\nu B_r}(\nu y) \\ &\leq f_{A_r}(x) \vee f_{B_r}(y)\ \forall\ x, y \in M,\ \lambda, \nu \in K \\ &= (f_A(x) \vee 1 - r) \vee (f_B(y) \vee 1 - r) \\ &\leq f_A(x) \vee f_B(y) \end{aligned}$$

Hence the proof. □

Definition 3.4. Let $(A_r)_i,\ i \in J$ be an arbitrary non empty family of U^{G_M}, then

(1) $\bigcap_{i \in J}(A_r)_i = \{x, t_{\bigcap_{i \in J}(A_r)_i}(x), i_{\bigcap_{i \in J}(A_r)_i}(x), f_{\bigcap_{i \in J}(A_r)_i}(x) : x \in M\}$ where

$$t_{\bigcap_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\wedge} t_{(A_r)_i}(x)$$
$$i_{\bigcap_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\wedge} i_{(A_r)_i}(x)$$
$$f_{\bigcap_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\vee} f_{(A_r)_i}(x)$$

(2) $\bigcup_{i \in J}(A_r)_i = \{x, t_{\bigcup_{i \in J}(A_r)_i}(x), i_{\bigcup_{i \in J}(A_r)_i}(x), f_{\bigcup_{i \in J}(A_r)_i}(x) : x \in M\}$ where

$$t_{\bigcup_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\vee} t_{(A_r)_i}(x)$$
$$i_{\bigcup_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\vee} i_{(A_r)_i}(x)$$
$$f_{\bigcup_{i \in J}(A_r)_i}(x) = \underset{i \in J}{\wedge} f_{(A_r)_i}(x)$$

Theorem 3.2. Let $(A_r)_i,\ i \in J$ be an arbitrary non empty family of U^{G_M}, then $\lambda(\bigcup_{i \in J}(A_r)_i) = \bigcup_{i \in J}(\lambda(A_r)_i)\ for\ \lambda \in K$

Proof: Consider $\lambda\bigcup_{i\in J}(A_r)_i = \{x, t_{\lambda\bigcup_{i\in J}(A_r)_i}(x), i_{\lambda\bigcup_{i\in J}(A_r)_i}(x), f_{\lambda\bigcup_{i\in J}(A_r)_i}(x) : x\in M,\ \lambda\in K\}$
Now

$$\begin{aligned} t_{\lambda\bigcup_{i\in J}(A_r)_i}(x) &= \begin{cases}\vee\{t_{\bigcup_{i\in J}(A_r)_i}(y)\} & if\ y\in M, x=\lambda y\\ 0 & otherwise\end{cases}\\ &= \begin{cases}\vee\{\underset{i\in J}{\vee} t_{(A_r)_i}(y)\} & if\ : y\in M, x=\lambda y\\ 0 & otherwise\end{cases}\\ &= \underset{i\in J}{\vee} t_{\lambda(A_r)_i}(x)\\ &= t_{\bigcup_{i\in J}\lambda(A_r)_i}(x)\end{aligned}$$

Similarly $i_{\lambda\bigcup_{i\in J}(A_r)_i}(x) = i_{\bigcup_{i\in J}\lambda(A_r)_i}(x)$
Now

$$\begin{aligned} f_{\lambda\bigcup_{i\in J}(A_r)_i}(x) &= \begin{cases}\wedge\{f_{\bigcup_{i\in J}(A_r)_i}(y)\} & if\ y\in M, x=\lambda y\\ 1 & otherwise\end{cases}\\ &= \begin{cases}\wedge\{\underset{i\in J}{\wedge} f_{(A_r)_i}(y)\} & if\ y\in M, x=\lambda y\\ 1 & otherwise\end{cases}\\ &= \underset{i\in J}{\wedge} f_{\lambda(A_r)_i}(x)\\ &= f_{\bigcup_{i\in J}\lambda(A_r)_i}(x)\end{aligned}$$

Hence $\lambda(\bigcup_{i\in J}(A_r)_i) = \bigcup_{i\in J}(\lambda(A_r)_i)\ for\ \lambda\in K$ □

Theorem 3.3. Let $A_r \in U^{G_M}$. Then A_r hold the following:

(1) $\hat{N}_{\{0\}} \subseteq A_r$
(2) $\lambda A_r \subseteq A_r\ \forall\lambda\in K$
(3) $A_r + A_r \subseteq A_r$

Proof: Consider $P\in U(M)$

1. We know $\hat{N}_{\{0\}}(x) = \{(x, t_{\hat{N}_{\{0\}}}(x), i_{\hat{N}_{\{0\}}}(x), f_{\hat{N}_{\{0\}}})(x) : x\in M\}$ where

$$\hat{N}_{\{0\}}(x) = \begin{cases}(1,1,0) & x=0\\ (0,0,1) & x\neq 0\end{cases}$$

Then obviously, $t_{\hat{N}_{\{0\}}}(x) \le t_{A_r}(x),\ i_{\hat{N}_{\{0\}}}(x)\le i_{A_r}(x)\ and\ f_{\hat{N}_{\{0\}}}(x)\ge f_{A_r}(x)\ \forall\ x\in M$
Hence $\hat{N}_{\{0\}}\subseteq A_r$

2. Consider $\lambda A_r = \{x, t_{\lambda A_r}(x), i_{\lambda A_r}(x), f_{\lambda A_r}(x) : x\in M\}$ where

$$\begin{aligned} t_{\lambda A_r}(x) &= \begin{cases}\vee\{t_{A_r}(y)\} & if\ y\in M, x=\lambda y\\ 0 & otherwise\end{cases}\\ &\le t_{A_r}(x)\ \forall x\in M\ [t_{A_r}(x) = t_{A_r}(ry)\ge t_{A_r}(y)]\end{aligned}$$

Similarly $i_{\lambda A_r}(x)\le i_{A_r}(x),\ f_{\lambda A_r}(x)\ge f_{A_r}(x), \forall x\in M.$
Hence $\lambda A_r\subseteq A_r$.

3. Consider $x\in M, \lambda\in K$

$$\begin{aligned} t_{A_r+A_r}(x) &= \vee\{t_{A_r}y\wedge t_{A_r}(z) : y,z\in M, x=y+z\}\\ &\le t_{A_r}(x)\ \forall x\in M\end{aligned}$$

Similarly we can prove, $i_{A_r+A_r}(x)\le i_{A_r+A_r}(x)\ and\ f_{A_r+A_r}(x)\ge f_{A_r}(x)$
Therefore $A_r + A_r\subseteq P$ □

Corollary 3.3.1. Let $A_r\in U^{G_M}$, then A_r satisfy the following:

(1) $\lambda A_r + \nu A_r \subseteq A_r,\ \forall\lambda,\nu\in K$

4. Conclusion

The special class of neutrosophic sets and their algebraic study advance the application of neutrosophic sets in signal systems and decision making. Structure-preserving properties can be easily derived from r-neutrosophic G-submodules, allowing for the analysis of characteristics in uncertain and indeterminate systems. Future research will focus on homomorphisms and isomorphisms of r-neutrosophic G-submodules, as well as tensor analysis.

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